Anti-Modules

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WOMoCoE 2016

Introduction

- Modules tell us about the organization of an ontology through the relationships between its logical subtheories.
- Modules also constrain how we can extend an ontology.

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- In general, an ontology is not equivalent to the union of its modules; there will be axioms that are not contained in any module of the ontology.
- Since almost all of the research within applied ontology has focused on the identification and extraction of the modules of an ontology, it has neglected to investigate such sentences which cannot be contained in any module.

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Definition: Residue

Let $T_1, \ldots, T_n$ be all proper modules of a theory T.
The residue R of T is the subtheory of T that is logically equivalent to

$$T \setminus (T_1 \cup \ldots \cup T_n).$$

Outline

- Examples of Residues
- Properties of Residues
- Application of Residues in Composition and Decomposition

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  - The other module of $T_{dolce\_present}$ consists of the axioms:

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- In addition, there are two axioms which are <u>not</u> contained in any module of $T_{dolce\_present}$ (the residue):

$$(\forall x,t,t_1)\, PRE(x,t) \wedge P(t_1,t) \supset PRE(x,t_1),$$

$$(\forall x,t,t_1,t_2)\, PRE(x,t_1) \wedge PRE(x,t_2) \wedge SUM(t,t_1,t_2) \supset PRE(x,t).$$

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$$before(x, y) \wedge before(y, z) \supset before(x, z).$$
$$(\forall x)\ timepoint(x) \supset \neg before(x, x). \quad (2)$$
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- In each of these timepoint ontologies, the residue is equivalent to the entire ontology.

Residues

Theorem 1: Residues

If R be the residue of a theory T, then $\Sigma(R) = \Sigma(T)$.

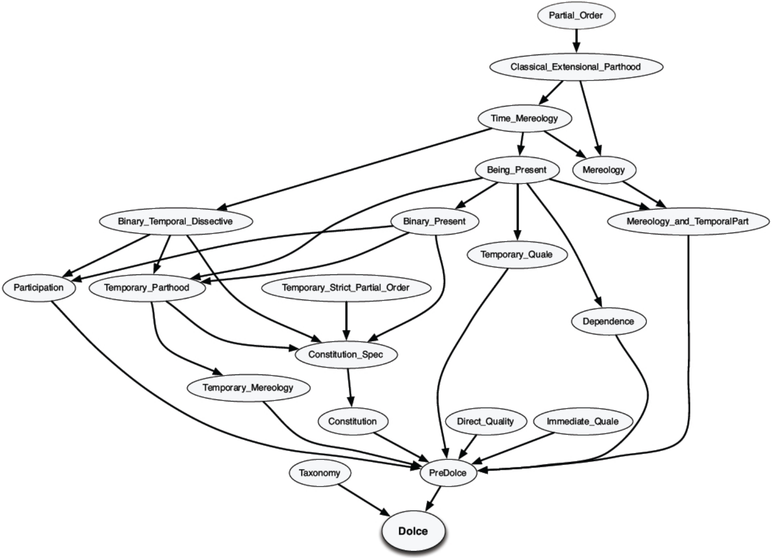
Residues & Ontology Decomposition

Ontology decomposition is the problem of finding a set of modules $T_1, ..., T_n$ and residue R for an ontology T such that

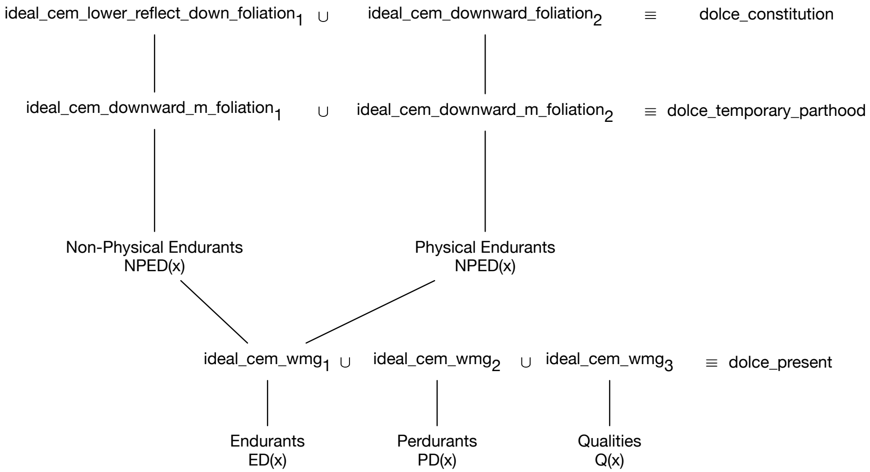
$$T = T_1 \cup ... \cup T_n \cup R$$

In a perfectly modularized ontology, $R = \emptyset$.

Residues & Ontology Decomposition: DOLCE (from [2])



Residues & Ontology Decomposition: DOLCE (from [1])



Residues & Ontology Composition

A common practice in ontology design is to identify existing ontologies that satisfy parts of the ontological commitments and requirements, and reuse them as building modules of the new ontology.

However, often there are ontological commitments that cannot be captured by the modules alone, and residues are needed to properly axiomatize such commitments.

Residues & Ontology Composition

- The $\mathbb{H}^{subposet}$ Hierarchy is a collection of mathematical ontologies which are developed for verification of spatial and temporal ontologies. The weakest ontology in this hierarchy is constructed by taking the weakest mereology and the weakest axiomatization of orderings.
- Ontologies in this hierarchy are used in the verification of:
  - Periods (van Bentham)
  - Subactivity in the PSL Ontology
  - Multimereology

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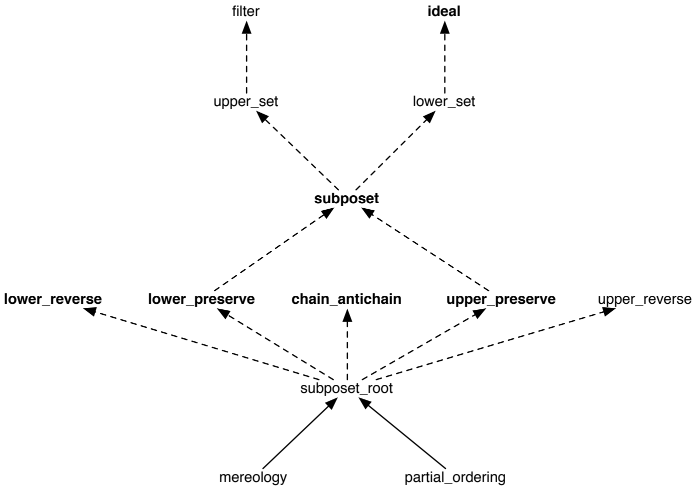

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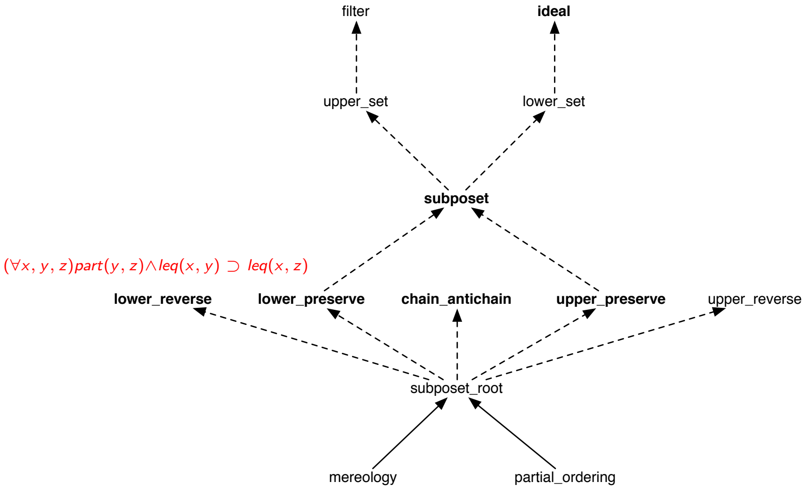

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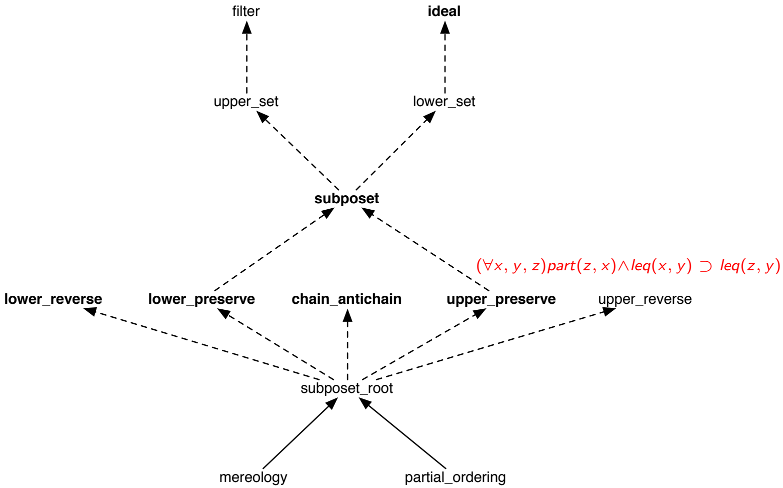

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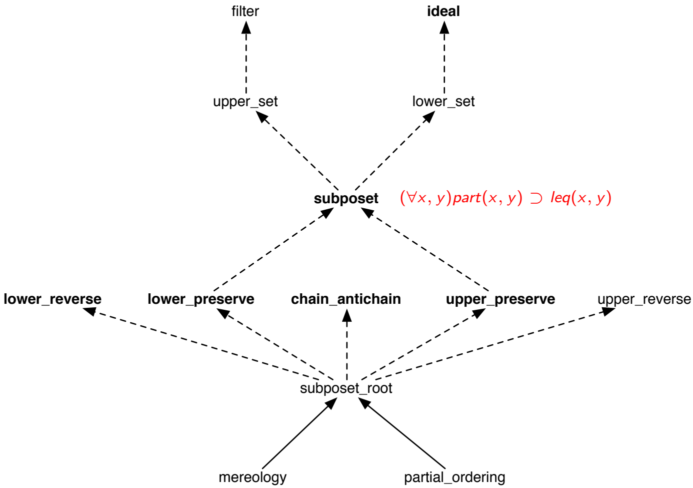

Residues & Ontology Composition

An example of an ontology from $\mathbb{H}^{subposet}$ Hierarchy is $T_{subposet}$ itself. $T_{subposet}$ is obtained by extending $T_{lower\_preserve} \cup T_{upper\_preserve}$ with the residue axiom:

$$(\forall x, y)\, part(x, y) \supset leq(x, y)$$

Residues & Ontology Composition

Design Principle

Consider the root theory T of an ontology hierarchy so that T contains weak reductive modules $T_1, ..., T_n$ and residue R. Other ontologies in the hierarchy are obtained by strengthening at least one of $T_1, ..., T_n$, or strengthening R.

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- $T_{periods\_root}$ is weakly reducible to $T_{prod\_mereology}$ and $T_{partial\_ordering}$, so that we can decompose $T_{periods}$ into two modules with the residue:

$$(\forall x, y, z)\ precedence(x, y) \wedge inclusion(z, x) \supset precedence(z, y)$$

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- Alternatively, we can specify new time period ontologies by extending the same modules with different residues. For example, we can extend $T_{periods\_root}$ with the residue:

$$(\forall x, y, z)\ precedence(x, y) \wedge inclusion(z, y) \supset precedence(x, z)$$

Summary

- We have introduced the notion of the residue of a theory, which is the set of sentences not contained in any module of the theory.
- Residues play a critical role in the design and verification of ontologies:
  - For verification, residues are used to distinguish between reductive modules and weak reductive modules of a theory.
  - For design, the residue constrains how models of the theory can be constructed from models of its modules.

References & Additional Links I

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Axiomatized Relationships between Ontologies.
Master's thesis, University of Toronto, 2013.

KUTZ, O., AND MOSSAKOWSKI, T.
A Modular Consistency Proof for DOLCE.
In *Proceedings of the Twenty-Fifth AAAI Conference on Artificial Intelligence (AAAI 2011), August 7-11, 2011* (San Francisco, California, USA, 2011), W. Burgard and D. Roth, Eds., AAAI Press, pp. 227–234.

Proof for Theorem #1

Proof.

Let R be the residue of T. By definition, R is a subtheory of T, so $\Sigma(R) \subseteq \Sigma(T)$.

Suppose, for a contradiction, that $\Sigma(R) \neq \Sigma(T)$. Then $\Sigma(R)$ must be a proper subset of $\Sigma(T)$. Since $\Sigma(R) \subset \Sigma(T)$, there must be a module T' of T which contains sentences with the signature $\Sigma(R)$. Since T' is a module of T, T has to be a conservative extension of T'. This means that any sentence in $T \setminus T'$ extends the signature of T' (otherwise T would be a non-conservative extension of T').

Thus, R must be included in T', which is a contradiction. $\square$